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A Note On The Partition Dimension of Thorn of Fan Graph

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Abstrak: Misalkan G adalah suatu graf terhubung.Himpunan titikV(G) di partisi menjadi k buah partisi S_1, S_2,…, S_k yang saling lepas. Notasikan $\Pi = \{S_1, S_2, ..., S_k\}$.Maka representasi v ∈V(G) terhadap phi didefenisikan : $r(v|\Pi)=(d(v,S_1),d(v,S_2),...,d(v,S_k))$, Jika untuk setiap dua titik yang berbeda $u, v \in$ V(G) berlaku $r(u|\Pi) = r(v|\Pi)$, maka Π dikatakan partisi penyelesaian dari graf G. Graf kipas diperoleh dari operasi graf hasil tambah K_1+P_n. Graf kipas dinotasikan dengan $F_{1,n}$ untuk n ≥ 2. Graf thorn untuk graf kipas diperoleh dengan cara menambahkan daun sebanyak l_i kesetiap titik di graf kipas, dinotasikan dengan $Th(F_{1,n}, l_1, l_2, \ldots, l_{n+1})$. Pada tulisan ini, akan dibahas tentang dimensi partisi graf thorn dari graf kipas $F_{1,n}$untuk n = 2, 3,4.

Kata kunci: *Partisi penyelesaian, dimensi partisi, graf kipas, graf thorn*

Abstract: Let $G = (V, E)$ be a connected graph and $S \subseteq V(G)$. For a vertex $v \in V(G)$ and an ordered *k*-partition $\Pi = \{S_1, S_2, \ldots, S_k\}$ of $V(G)$, the presentation of v concerning Π is the *k*-vector $r(v|\Pi) = (d(v, S_1), d(v, S_2), \ldots, d(v, S_k))$, where $d(v, S_i)$ denotes the distance between v and S_i for $i \in \{1,2, \ldots, n\}$. The *k*-partition Π is said to be resolving if for every two vertices $u, v \in V(G)$, the representation $r(u|\Pi) \neq r(v|\Pi)$. The minimum k for which there is a resolving *k*-partition of $V(G)$ is called the partition dimension of G, denoted by $pd(G)$. Let $V(G) = \{x_1, x_2, \ldots, x_n\}$. Let $l_1, l_2, \ldots, l_n$ be non-negative integer, $l_i \geq 1$,for $i \in \{1,2, \ldots, n\}$. The thorn of G, with parameters $l_1, l_2, \ldots, l_n$ is obtained by attaching l_i vertices of degree one to the vertex x_i, denoted by $Th(G, l_1, l_2, \ldots, l_n)$. In this paper, we determine the partition dimension of $Th(G, l_1, l_2, \ldots, l_n)$where $G \simeq F_{1,n}$, the fan on n+1 vertices, for $n = 2,3,4$.

Keywords*: Resolving partition, partition dimension, fan, thorn graph*

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1. Introduction

Let $G = (V, E)$ be an arbitrary connected graph. [1] defined the partition dimension as follows. Let u and v be two vertices in $V(G)$. The distance $d(u, v)$ is the length of the shortest path between u and $v$in G. For an ordered set $\Pi = \{S_1, S_2, \ldots, S_k\}$ of vertices in a connected graph G and a vertex $v$of G, the k-vector $r(v|\Pi) = (d(v, S_1), d(v, S_2), \ldots, d(v, S_k))$, is the presentation of v with respect to Π. The minimum k for which there is a resolving k-partition of $V(G)$ is called the partition dimension of G, denoted by $pd(G)$. All notation in graph theory needed in this paper refers to [2].

Stated the following theorem.

Theorem 1.1. [2] *Let G be a connected graph on n vertices, $n \geq 2$. Then $pd(G) = 2$ if and only if $G \simeq P_n$.*

In the same paper, Chartrand et al. [2] also gave the necessary condition in partitioning the set of vertices as follows.

Lemma 1.2. [2] *Suppose that Π is the resolving partition of $V(G)$and $u, v \in V(G)$. If $d(u, w) = d(v, w)$ for every vertex $w \in V(G) \backslash \{u, v\}$ then u and v belong to a different class of* Π.

2. Main Results

The fan $F_{1,n}$ on $n + 1$ vertices is defined as the graph constructed by joining K_1 and P_n, denoted by $K_1 + P_n$ where K_1 is the complete graph on 1 vertex and P_n Is a path on n vertices, for $n \geq 2$. The vertex set and edge set of $F_{1,n}$ are as follows.

$$V(F_{1,n}) = \{x_i | 1 \leq i \leq n + 1\},$$
$$E(F_{1,n}) = \{x_1 x_t | 1 \leq t \leq n\} \cup \{x_s x_{s+1} | 1 \leq s \leq n - 1\}.$$

$l_1, l_2, \ldots, l_{n+1}$Be some positive integer. The thorn graph of $F_{1,n}$ is obtained by adding l_i leaves to vertex x_i, for $1 \leq i \leq n + 1$, denoted by $Th(F_{1,n}, l_1, l_2, \ldots, l_{n+1})$. The construction of thorn graph is taken from [3]. The vertex set and edge set of $H \simeq Th(F_{1,n}, l_1, l_2, \ldots, l_{n+1})$ are as follows.

$V(H) = \{x_i | 1 \leq i \leq n + 1\} \cup \{x_{ij} | 1 \leq i \leq n + 1,\ 1 \leq j \leq l_i\}$, and
$E(H) = \{x_1 x_t | 1 \leq t \leq n\} \cup \{x_s x_{s+1} | 1 \leq s \leq n - 1\} \cup \{x_i x_{ij} | 1 \leq i \leq n, 1 \leq j \leq n\}$.

In Theorem 2.1 we determine the partition dimension of $Th(F_{1,2}, l_1, l_2, l_3)$for $l_i \geq 1$, $i \in 1,2,3$.

Theorem 2.1. Let $Th(F_{1,2}, l_1, l_2, l_3)$ be thorn of fan $F_{1,2}$with $l_i \geq 1,\ i \in 1,2,3$. Denote $l_{max} = \max\{l_1, l_2, l_3\}$.
The partition dimension of $Th(F_{1,2}, l_1, l_2, l_3)$ is

$$pd(Th(F_{1,2}, l_1, l_2, l_3)) = \begin{cases} 3, & for\ l_{max} = 1, 2\ or\ 3 \\ l_{max}, & for\ l_{max} \geq 4 \end{cases}$$

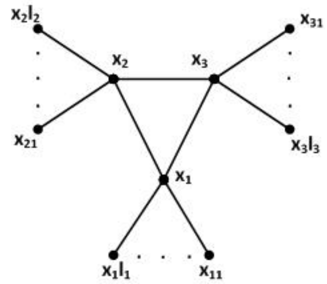

Figure 1. $Th(F_{1,2}, l_1, l_2, l_3)$

Proof. The proof is divided into two cases.

Case 1. $1 \leq l_{max} \leq 3$.
Let $H_1 \simeq Th(F_{1,2}, l_1, l_2, l_3)$, with $1 \leq l_{max} \leq 3$. Because $H_1 \neq P_n$ then from Theorem 1.1, it is obtained that $pd(H_1) \geq 3$. Next, it will be shown that $pd(H_1) \leq 3$ by constructing three ordered partitions. Note that from Lemma 1.2, every leaf at the vertex x_i Must be on a different partition. Therefore, we define $\Pi = \{S_1, S_2, S_3\}$, where
$S_i = \{x_i, x_{ki} | 1 \leq i \leq 3,\ 1 \leq k \leq 3\}$,
Because of $d(v, S_i) = 0$ while $d(u, S_i) \neq 0$ for $v \in S_i$and $u \notin S_i$, it is clear that every two vertices in different partitions have different representations. Therefore, it is sufficient to check the representations of two vertices in the same partition. Because of $d(x_{ki}, S_j) = d(x_i, S_j) + 1$for $i \neq j$, $1 \leq i, j \leq 3$, then $r(x_{ki}|\Pi) \neq r(x_i|\Pi)$. Thus, we have that $pd(H_1) \leq 3$.

Case 2. $l_{max} \geq 4$.
Let $H_2 \simeq Th(F_{1,2}, l_1, l_2, l_3)$, with $l_{max} \geq 4$. Let $l_{max} = m$ and suppose that $pd(H_2) = m - 1$. Then we have $\Pi = \{S_1, S_2, \ldots, S_{m-1}\}$. Thus there are at least two vertices, namely $x_{1p}$and x_{1q}, in the same partition, for $1 \leq p,\ q \leq m$. But from Lemma 1.2, x_{1p} and x_{1q} Must be placed in different partitions. Therefore, $|\Pi| \geq m$, a contradiction.

Next, we construct $\Pi = \{S_1, S_2, \ldots, S_m\}$, where
$S_i = \{x_i, x_{ki} | 1 \leq i \leq 3,\ 1 \leq k \leq 3\}$,
$S_j = \{x_{kj} | 1 \leq k \leq 3,\ 4 \leq j \leq l_{max}\}$,
Because of $d(x_{ki}, S_j) = d(x_i, S_j) + 1$for $i \neq j$, $1 \leq i,\ j \leq l_{max}$, then $r(x_{ki}|\Pi) \neq r(x_i|\Pi)$. Next, because of $d(x_{ki}, S_j) \neq d(x_{li}, S_j) + 1$for $k \neq l$, $1 \leq k,\ l \leq l_{max}$, it is clear that $r(x_{ki}|\Pi) \neq r(x_{li}|\Pi)$. Therefore, we have $pd(H_2) \leq l_{max}$. ■

In Theorem 2.2 we determine the partition dimension of $Th(F_{1,3}, l_1, l_2, l_3, l_4)$for $l_i \geq 1$, $i \in 1,2,3,4$.

Theorem 2.2. Let $Th(F_{1,3}, l_1, l_2, l_3, l_4)$ be a thorn of fan $F_{1,3}$with $l_i \geq 1$, $i \in 1,2,3,4$. Denote $l_{max} = \max\{l_1, l_2, l_3, l_4\}$. Let x_{l_i} be the vertex in $F_{1,3}$ with l_i leaves, and $|x_{lmax}|$ be the number of vertices with l_{max} Leaves. The partition dimension of $Th(F_{1,3}, l_1, l_2, l_3, l_4)$ is

$$pd(Th(F_{1,3}, l_1, l_2, l_3, l_4)) = \begin{cases} 3, \ for \ l_{max} = 1 \ or \ 2, \\ \quad and \ for \ l_{max} = 3, \ \ if \ \ |x_{lmax}| = 1 \ or \ 2, \\ 4, \ for \ l_{max} = 3, \ \ if \ \ |x_{lmax}| = 3 \ or \ 4, \\ l_{max}, \ \ for \ l_{max} \geq 4. \end{cases}$$

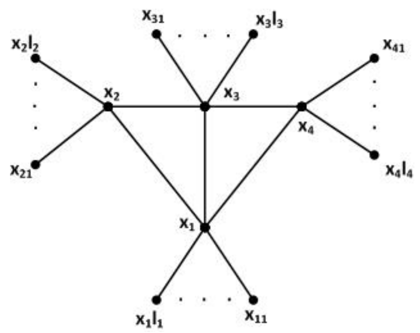

Figure2. $Th(F_{1,3}, l_1, l_2, l_3, l_4)$

Proof. The proof is similar to the proof of Theorem 2.1 ■

In Theorem 2.3 we determine the partition dimension of $Th(F_{1,4}, l_1, l_2, l_3, l_4, l_5)$for $l_i \geq 1, \ i \in 1,2,3,4,5$.

Theorem 2.3. Let $Th(F_{1,4}, l_1, l_2, l_3, l_4, l_5)$ be a thorn of fan $F_{1,4}$with $l_i \geq 1, i \in 1,2,3,4,5$. Denote $l_{max} = \max\{l_1, l_2, l_3, l_4, l_5\}$. Let x_{l_i} be the vertex in $F_{1,4}$ with l_i leaves, and $|x_{lmax}|$ be the number of vertices with l_{max} Leaves. The partition dimension of $Th(F_{1,3}, l_1, l_2, l_3, l_4, l_5)$ is

$$pd(Th(F_{1,4}, l_1, l_2, l_3, l_4, l_5)) = \begin{cases} 3, \ for \ l_{max} = 1, \\ \quad and \ for \ l_{max} = 2, \ \ if \ \ |x_{lmax}| = 1 \ or \ 2, \\ \quad and \ for \ l_{max} = 3, \ \ if \ \ |x_{lmax}| = 1, \\ 4, \ for \ l_{max} = 2, \ \ if \ \ |x_{lmax}| = 3, 4 \ or \ 5, \\ \quad and \ for \ l_{max} = 3, \ \ if \ \ |x_{lmax}| = 2, 3, 4 \ or \ 5, \\ \quad and \ for \ l_{max} = 4, \ \ if \ \ |x_{lmax}| = 1, 2, 3 \ or \ 4, \\ 5, \ for \ l_{max} = 4, \ \ if \ \ |x_{lmax}| = 5, \\ l_{max}, \ \ for \ l_{max} \geq 5 \end{cases}$$

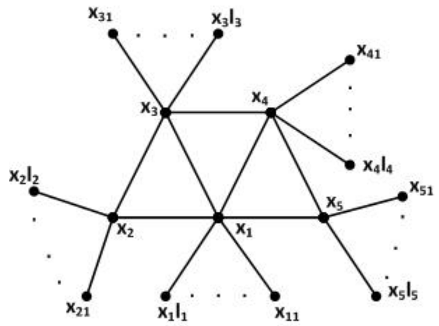

Figure 3. $Th(F_{1,4}, l_1, l_2, l_3, l_4, l_5)$

Proof. The proof is similar to the proof of Theorem 2.1 and Theorem 2.2.

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