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Rarity of Joint Probability Between Interest and Inflation Rates in the 1998 Economic Crisis in Indonesia and Their Comparison Over Three Time Periods

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Abstrak. Setelah lebih dari dua puluh tahun, tidak ada krisis ekonomi separah tahun 1998 berdasarkan inflasi dan suku bunga. Menarik untuk dibandingkan kondisi sebelum dan setelah krisis tahun 1998 serta kondisi ekonomi pada dekade terakhir di Indonesia. Oleh karena itu, penelitian ini bertujuan menganalisis hubungan antara tingkat inflasi dan suku bunga menggunakan distribusi bersama berbasis copula. Dari distribusi bersama tersebut, periode ulang bersama pada krisis ekonomi tahun 1998 diestimasi. Hasil penelitian menunjukkan bahwa copula Gumbel adalah fungsi copula bivariat yang paling cocok untuk membangun distribusi bersama antara tingkat inflasi dan suku bunga pada tahun 1990-2019 dengan dependensi ekor atas sebesar 0.6224. Periode ulang bersama antara tingkat inflasi dan suku bunga yang lebih parah daripada tahun 1998 secara bersamaan adalah 389 tahun dengan interval kepercayaan 95% yaitu [47, ∞] tahun. Hasil ini sangat tidak pasti karena banyak faktor yang mempengaruhi tingkat inflasi dan suku bunga. Tingkat inflasi mengalami penurunan pada periode setelah krisis 1998. Pada dekade terakhir, tingkat inflasi dan suku bunga jauh lebih rendah dibandingkan dua periode sebelumnya.

Keywords:
Copula, Economic crisis, Inflation rate, Interest rate, Joint Return Period, Uncertainty

Abstract. After more than twenty years, there has been no economic crisis as severe as 1998 based on inflation and interest rates. It is interesting to compare the conditions before and after the 1998 crisis and the economic conditions in the last decade in Indonesia. Therefore, this study aims to analyze the relationship between inflation and interest rates using a copula-based joint distribution. The joint return period of the 1998 economic crisis is estimated from this joint distribution. The results showed that the Gumbel copula is the most suitable bivariate copula to construct a joint distribution between inflation and interest rates in 1990-2019, with an upper tail dependency of 0.6224. Moreover, the joint return period between inflation and interest rates more severe than 1998 is 389 years with a 95% confidence interval of [47, ∞] years. This result is uncertain because many factors affect inflation and interest rates. The inflation rate decreased after the 1998 crisis. Meanwhile, in the last decade, the inflation and interest rates were much lower than in the two previous periods.

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1. Introduction

The economic crisis in 1998 played an essential role in global history, including in Indonesia [1]. The devaluation of the Indonesian currency (Rupiah) causes inflation and reduces actual public spending on health. Household spending on health also declined, in absolute terms and as a percentage of overall expenditure. Self-reported morbidity increased sharply from 1997 to 1998 in Indonesia's rural and urban areas [2]. The crisis led to a substantial reduction in health care utilization during the same period, as the proportion of household survey respondents who reported an illness or injury seeking care from a modern health care provider decreased by 25% [3].

Several economic indicators can investigate the rarity of the economic crisis in 1998, e.g., inflation and interest rates. We are focusing on the inflation rate in the health sector due to increased morbidity in 1998 but low utilization of health facilities. The inflation and rising interest rates affect the decline in people's purchasing power, slowing the economy and pushing it toward a recession [4]. After more than twenty years, there was no economic crisis as severe as in 1998 based on inflation and interest rates (Fig. 1). Moreover, very few experts research the joint return period between inflation and interest rates during the 1998 economic crisis in Indonesia using copula.

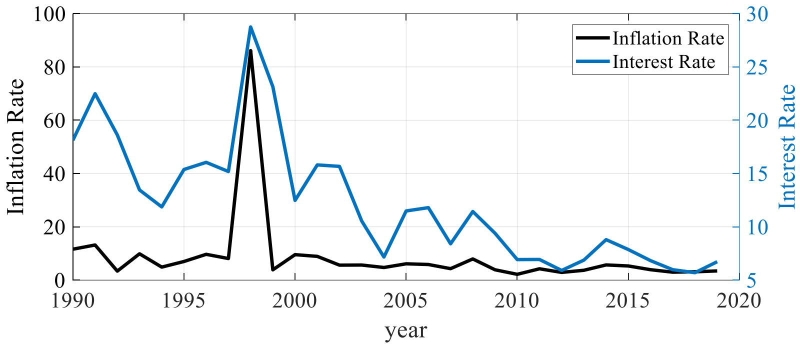

Figure 1. Inflation rates in the health category and Bank of Indonesia (BI) interest rates

One popular approach to identifying the relationship between two random variables is copula-based joint distribution [5]. Copula provides a simple way to construct the joint distribution of two variables [6-7]. Therefore, we can calculate the joint probability of a specific event, e.g., the 1998 economic crisis [8]. Moreover, we also can estimate the rarity and joint return period of this specific event using the joint probability [9].

This study uses copula-based joint distribution to observe the relationship between inflation and interest rates from 1990 to 2019. We estimate the joint return period of the 1998 economic crisis from the joint distribution. Therefore, the result can assess the severity of the 1998 economic crisis and estimate when similar events will occur again. We divide the data into three periods, i.e., before the 1998 crisis, after the 1998 crisis, and after the 2008 crisis. Using the same approach, we calculate and compare the joint distribution of these three periods.

2. Materials and Methods

2.1 Materials

We use two types of data, i.e., inflation and interest rates. We use interest rates in the health category from the Ministry of Trade Republic of Indonesia in 1990-2019 (https://statistik.kemendag.go.id/inflation), while the interest rate of time deposit in Rupiah obtained from the Bank of Indonesia in 1990-2019 (https://www.bi.go.id/seki/). The data has an annual period and shows the highest value is in 1998 (Fig. 1). To estimate the joint return period of the 1998 economic crisis, we use all data from 1990 to 2019. After that, we divide the data into three periods, i.e., 1990-1998, 1999-2008, and 2009-2019.

2.2 Copula Function

Let X_1 and X_2 are representing inflation and interest rates, respectively. For bivariate cases (X_1, X_2), the copula function links the multivariate distribution $F_X(x_1, x_2)$ to their univariate marginal distributions $F_1(x_1)$ and $F_2(x_2)$, given by

$$F_X(x_1, x_2) = C[F_1(x_1), F_2(x_2)] \tag{1}$$

where $C: [0,1] \times [0,1] \rightarrow [0,1]$ called as copula function [10][11]. By differencing the left and right-hand sides, we get the joint probability density function between X_1 and X_2, i.e.,

$$f_X(x_1, x_2) = \frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2} C[F_1(x_1), F_2(x_2)] = c[F_1(x_1), F_2(x_2)] \cdot f_1(x_1) \cdot f_2(x_2) \tag{2}$$

where c is called a copula density function, while f_1 and f_2 are the probability density functions of X_1 and X_2 respectively [12][13][14]. To capture different upper and lower tail behaviour, we use copula functions from the Archimedean family (Table 1), such as Gumbel, Frank, and Clayton [15][16][17]. The Clayton (Gumbel) copula has lower (upper) tail dependence, but the Frank copula has no tail dependence.

Table 1. The selected bivariate copula function and their properties

Copula name	Copula function	Parameter range	Tail dependence Lower	Tail dependence Upper
Clayton	$(u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$	$\theta > 0$	$2^{-1/\theta}$	0
Gumbel	$\exp[-(w_1^{\theta} + w_2^{\theta})^{1/\theta}]$, $w_i = -\ln(u_i)$	$\theta \geq 1$	0	$2 - 2^{1/\theta}$
Frank	$-\frac{1}{\theta}\ln\left[1 + \frac{w_1 w_2}{(e^{-1} - 1)}\right]$, $w_i = e^{-\theta u_i} - 1$	$\theta \neq 0$	0	0

u_i are the transformed variables $u_i = F_i(x_i)$ for $i = 1,2$

2.3 Estimating The Copula Parameters

To estimate the parameters of copulas, we use the inference of functions for margins (IFM) method [18][19] and the fittest copula chosen based on Root Mean Squared Error (RMSE), Akaike's Information Criterion (AIC), and Kolmogorov-Smirnov Error (KSE) [20]. In principle, IFM is a two-step method to estimate the copula parameters. The first step of IFM is to estimate the marginal distributions of each variable. In this paper, we use lognormal and generalized extreme value distribution to fit the marginal distribution of data. The Probability Density Function (PDF) and the Cumulative Distribution Function (CDF) of the lognormal (LN) distribution are given by

$$f_{LN}(x \mid \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[\frac{-(\ln x - \mu)^2}{2\sigma^2}\right], x > 0 \tag{3}$$

and

$$F_{LN}(x \mid \mu, \sigma) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right) \tag{4}$$

where $\Phi(x)$ is the CDF of standard normal distribution, μ is the location parameter, and σ is the scale parameter [21]. Meanwhile, the PDF and CDF of the Generalized Extreme Value (GEV) distribution are given by

$$f_{GEV}(x \mid k, \mu, \sigma) = \frac{1}{\sigma} \exp\left[-\left(1 + k\frac{x - \mu}{\sigma}\right)^{-\frac{1}{k}}\right]\left(1 + k\frac{x - \mu}{\sigma}\right)^{-1-\frac{1}{k}} \tag{5}$$

and

$$F_{GEV}\left(x \mid k,\mu,\sigma\right)=\exp\left[-\left(1+k\frac{x-\mu}{\sigma}\right)^{-\frac{1}{k}}\right] \tag{6}$$

for $x \in R$, and $k \neq 0$ [22][23]. We employed the Anderson-Darling statistical test with a 5% significance level to test the goodness-of-fit of distribution to the actual data [24].

Using these marginal distributions, we fit the copula parameter by maximizing the log of the likelihood function, i.e.,

$$\begin{aligned}\hat{\theta} &= \arg\max \ln L = \arg\max \ln \prod_{t=1}^{N} c_X\left(F_1(x_1^t;\hat{\alpha}_1), F_2(x_2^t;\hat{\alpha}_2);\theta\right) \\ &= \arg\max \sum_{t=1}^{N} \ln c_X\left(F_1(x_1^t;\hat{\alpha}_1), F_2(x_2^t;\hat{\alpha}_2);\theta\right)\end{aligned} \tag{7}$$

where $\hat{\alpha}$ is the estimate of marginal distribution parameters and $\hat{\theta}$ is the estimate of the copula parameter [25].

2.4 Joint Return Period

From the joint distribution, we can estimate the probability (P) of a condition exceeding a certain critical multivariate threshold. In this case, we use inflation and interest rates at the 1998 economic crisis as a threshold. Therefore, the probability of two events coinciding as severe than in 1998 is defined by [26]

$$P_{AND}=P(X>x \cap Y>y)=1-F_X(x)-F_Y(y)+C\left(F_X(x),F_Y(y)\right) \tag{8}$$

where x and y are inflation and interest rates in 1998, respectively. For the limited data, we can estimate the 95% confidence interval for P, i.e., $P \pm \varepsilon$ with

$$\varepsilon = 2\sqrt{\frac{P(1-P)}{N}} \tag{9}$$

where N is the sample size of data [27]. Since we use annual data, the joint return period (in years) is defined by $1/P$ [9].

3. Results and Discussion

3.1 Fitting Processes

Table 2 shows the fitting result of the marginal distributions of the inflation and interest rates. After that, the copula parameters are estimated (Table 3) using the fittest marginal distribution. The statistical result shows that the chosen marginal distribution (Table 2) all passed the Anderson-Darling test with a 5% significance level, and the fittest copula is the Gumbel copula. The Gumbel copula is part of the extreme copula family and has an upper tail dependency (Table 1). Using the parameter value, the upper tail dependency between inflation and interest rates is

$$\lambda_U = 2-2^{1/2.1641} = 0.6224 \tag{10}$$

and the lower tail dependency is equal to zero.

Table 2. The fitting result of the marginal distribution of the inflation rate and the interest rate.

Data	Fittest distribution	p-value
The inflation rate	GEV ($k = 0.5253, \mu = 4.3886, \sigma = 2.0184$)	0.9856
The interest rate	Lognormal ($\mu = 2.3997, \sigma = 0.4529$)	0.7626

Table 3. The fitting result of the copula parameters between the inflation rate and the interest rate.

Copula	θ	KSE	RMSE	AIC
Gumbel	2.1641	0.12790	0.052504	-21.512
Frank	7.0248	0.13826	0.057042	-21.463
Clayton	1.6339	0.12823	0.058021	-15.910

3.2 Joint Probability Density Function

Using the fittest parameter, we got the marginal distributions (Eq. 11 and 12) and copula function (Eq. 13), i.e.,

$$F_1(x_1) = \exp\left[-\left(1+\frac{x_1-4.3886}{3.8424}\right)^{-\frac{1}{0.5253}}\right] \tag{11}$$

$$F_2(x_2) = \Phi\left(\frac{\ln x_2 - 2.3997}{0.4529}\right) \tag{12}$$

$$C_X(u_1,u_2) = \exp[-([-\ln(u_1)]^{2.1641} + [-\ln(u_2)]^{2.1641})^{1/2.1641}] \tag{13}$$

where $u_1 = F_1(x_1)$ and $u_2 = F_2(x_2)$. Therefore, we obtained the joint distribution between the inflation and interest rates using Eq. 1, i.e.,

$$\begin{aligned} F_X(x_1,x_2) &= C\left[F_1(x_1),F_2(x_2)\right] \\ &= \exp\left[-\left(\left\{-\ln\left[F_1(x_1)\right]\right\}^{2.1641} + \left\{-\ln\left[F_2(x_2)\right]\right\}^{2.1641}\right)^{1/2.1641}\right] \end{aligned} \tag{14}$$

where $\Phi(x)$ is the cumulative distribution function (CDF) of standard normal distribution. We need first the copula density function to construct the joint probability density function. By differencing the copula function to u_1 and u_2, we got the copula density function, i.e.,

$$\begin{aligned} c(u_1,u_2) &= \frac{\partial}{\partial u_1}\frac{\partial}{\partial u_2}C(u_1,u_2) = \frac{\partial}{\partial u_1}\frac{\partial}{\partial u_2}\exp\left[-\left\{\left[-\ln(u_1)\right]^{2.1641}+\left[-\ln(u_2)\right]^{2.1641}\right\}^{1/2.1641}\right] \\ &= \frac{1}{u_1}\frac{1}{u_2}\exp\left[-\left\{\left[-\ln(u_1)\right]^{2.1641}+\left[-\ln(u_2)\right]^{2.1641}\right\}^{\frac{1}{2.1641}}\right] \\ &\left[t-1+\left\{\left[-\ln(u_1)\right]^{2.1641}+\left[-\ln(u_2)\right]^{2.1641}\right\}^{\frac{1}{2.1641}}\right] \\ &\left[\left\{\left[-\ln(u_1)\right]^{2.1641}+\left[-\ln(u_2)\right]^{2.1641}\right\}^{\frac{1}{2.1641}-2}\right]\left[-\ln(u_1)\right]^{1.1641}\left[-\ln(u_2)\right]^{1.1641} \end{aligned} \tag{15}$$

where $u_1 = F_1(x_1)$ and $u_2 = F_2(x_2)$. Using Eq. 2, the joint probability density function between the inflation and interest rates is calculated and visualized using a contour plot (Fig. 2). The figure shows that the peaks of the joint probability between inflation and interest rates are 3.3 and 7.5, respectively. Meaning that during the last three decades, the value of inflation and interest rates mostly appears around 3.3 and 7.5. However, within those 30 years, there is an outlier data in 1998, which shows inflation and interest rates up to 86.14 and 28.75. Still, the probability of this event is minuscule which will be discussed in the following subchapter.

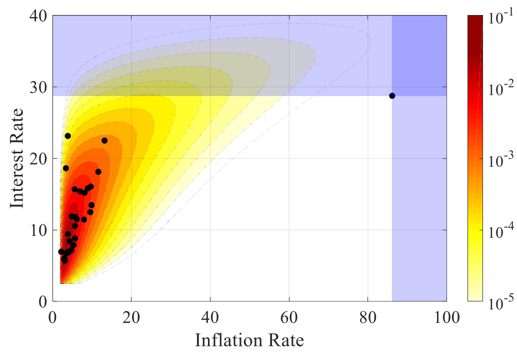

Figure 2. Joint probability density function between inflation and interest rates

3.3 Joint Return Period in 1998

The dark blue box on the right-top (Fig. 2) shows the hazard area when inflation and interest rates are simultaneously higher than in the 1998 economic crisis. Using Eq. 8, the probability of this area can calculate, i.e.,

$$\begin{aligned} P_{AND} &= P(X > 86.14 \cap Y > 28.75) \\ &= 1 - F_X(86.14) - F_Y(28.75) + C_X\left[F_X(86.14), F_Y(28.75)\right] = 0.26\% \end{aligned} \tag{15}$$

Thus, the return period of the hazard area is 1/0.0026 years or about 389 years. This means that the probability of inflation and interest rates higher than in 1998 simultaneously is 1/389 years. Using Eq. 9, the 95% confident interval of P_{AND} is [0,0.0211] so the 95% confident interval of return period is [47, ∞] years. This means that the condition of inflation and interest rates is worse than in 1998 is very rare, and there is a possibility that the 1998 economic crisis will not happen again. A wide 95% confidence interval indicates that the joint return period is uncertain due to many factors affecting inflation and interest rates.

3.4 Joint Return Period Over Three Periods

We calculate the joint probability density function between inflation and interest rates by three different periods using the same approach (Fig. 3).

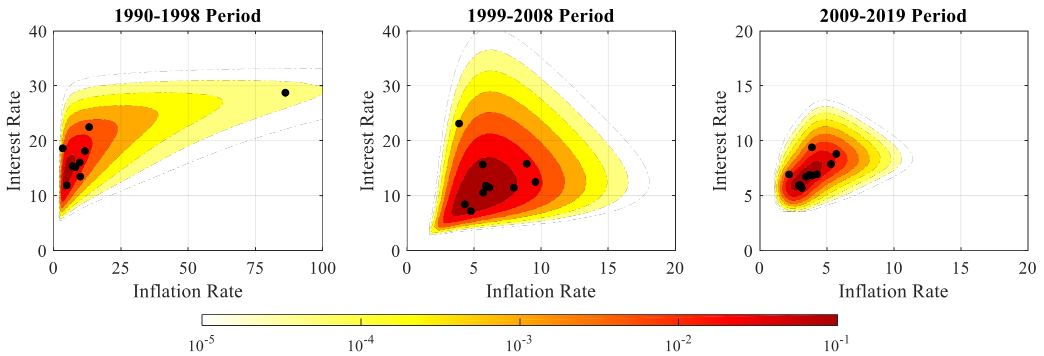

Figure 3. Joint probability density function between inflation and interest rates by three different periods

Fig. 3 shows that the joint probability density between inflation and interest rates decreased over three periods. From the first period (1990-1998), the joint probability density function in the second period (1999-2008) decreased the inflation rate but did not experience a significant decrease in the interest rate. On the other hand, in the last decade, the joint probability density function has decreased significantly in terms of inflation and interest rates.

4. Conclusion

The IFM method shows that the Gumbel copula is the fittest bivariate copula to construct the joint distribution between inflation and interest rates. From the joint distribution, the joint return period of inflation and interest rates higher than in 1998 simultaneously is 389 years with a 95% confident interval [47, ∞]. However, the result is uncertain due to many factors affecting inflation and interest rates. After dividing the data into three different periods, the joint probability density function has decreased significantly in terms of inflation and interest rates in the last decade compared to the previous two decades.

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